

## 11-4 Geometric Series

sum of a geometric sequence

$$2 + 6 + 18 + 54 + 162$$

$$r = 3$$

$$\begin{array}{r} S_5 = 2 + 6 + 18 + 54 + 162 \\ -3 \cdot S_5 = -6 - 18 - 54 - 162 - 486 \\ \hline -2S_5 = 2 - 486 \\ S_5 = \frac{2 - 486}{-2} = 242 \end{array}$$

$$S_n = \frac{a_1 - r a_n}{1 - r} \quad r \neq 1$$

$$S_n = \frac{a_1 - r a_1 r^{n-1}}{1 - r}$$

$$S_n = \frac{a_1 - r a_n}{1 - r}$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \quad r \neq 1$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Ex 1:  
Find the sum.  
 $a_1 = 7776$   
 $a_n = 6$

$$r = \frac{-1}{6}$$

$$S_n = \frac{7776 - (-\frac{1}{6})^6}{1 - (-\frac{1}{6})}$$

$$\frac{7777}{7/6} = 6,666$$

Ex 2:

$$a_1 = 2$$

$$S_8 = 765$$

$$r = 2$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$765 = \frac{a_1(1-2^8)}{1-2}$$

$$-765 = a_1(-255)$$

$$3 = a_1$$

Sigma Notation

$$\sum_{n=1}^{12} 3 \cdot 2^{n-1}$$

$$S_{12} = \frac{3 - 2 \cdot 6144}{1-2}$$

$$S_{12} = 12,285$$

$$a_1 = 3 \quad a_2 = 6$$

$$a_{12} = 6144$$

$$r = 2$$

Sigma Notation

$$\sum_{n=1}^5 6 \cdot \left(\frac{1}{3}\right)^{n-1} = 6 + 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$$

$$a_1 \cdot r^{n-1}$$

Write  
in expanded form

Put the following series into sigma notation.

$$48 + 24 + 12 + 6 + \dots + \frac{3}{8}$$

$$\sum_{n=1}^8 48 \cdot \left(\frac{1}{2}\right)^{n-1}$$

$48 \cdot \left(\frac{1}{2}\right)^{n-1} = \frac{3}{8}$   
 $\frac{1}{2}^{n-1} = \frac{1}{128}$   
 $\frac{1}{2}^{n-1} = \left(\frac{1}{2}\right)^7$   
 $n-1 = 7$   
 $n = 8$

Put the following series into sigma notation.

$$48 + 24 + 12 + 6 + \dots + \frac{3}{8}$$

$$\sum_{n=1}^8 48 \cdot \left(\frac{1}{2}\right)^{n-1}$$

$48 \cdot \left(\frac{1}{2}\right)^{n-1} = \frac{3}{8}$   
 $\frac{1}{2}^{n-1} = \frac{1}{128}$   
 $(h-1) \log \frac{1}{2} = \log \frac{1}{128}$   
 $h-1 = 7$   
 $n = 8$   
 $n-1 = 7$   
 $n = 8$

29. **HEALTH** Contagious diseases can spread very quickly. Suppose five people are ill during the first week of an epidemic and that each person who is ill spreads the disease to four people by the end of the next week. By the end of the tenth week of the epidemic, how many people have been affected by the illness?

$$a_1 = 5$$

$$a_2 = 20$$

$$r = 4$$

$$S_{10} = \frac{5(1 - 4^{10})}{1 - 4}$$

1,747,625

HW  
p597

15-27 odd, 30, 33, 35, 39, 41, 43, 47